INTERNATIONAL JOURNAL FOR NUMERICAL METHODS IN FLUIDS
Int. J. Numer. Meth. Fluids **31**: 261–274 (1999)

ESTIMATION OF BOUNDARY CONDITIONS FOR GROUND TEMPERATURE CONTROL USING KALMAN FILTER AND FINITE ELEMENT METHOD

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SUMMARY

This paper presents a numerical method in which the Kalman filter and the extended Kalman filter techniques are applied to the ground temperature control analysis with the finite element method. The purpose of this research is to identify the unknown parameters that are involved in the physical models and to estimate the temperature which can not be observed in a direct manner. In order to justify the present method, numerical computations are carried out, which ensures the adaptability of the method. Copyright © 1999 John Wiley & Sons, Ltd.

KEY WORDS: Kalman filter; extended Kalman filter; ground temperature control system; boundary condition; finite element method

1. INTRODUCTION

It is desirable that the lawns in sports stadiums are kept beautifully green. Sometimes, however, they are damaged by a change in temperature from daytime to night-time. The maintenance and management of lawns are not easy tasks, especially in the Japanese climate. Therefore, agricultural chemicals are sprinkled in large quantities on the sports fields in order to prevent the harmful insects that infect the lawns, and to keep the lawns green. However, pollution of the environment results because the agricultural chemicals eventually flow to the surrounding rivers and underground water.

To solve this serious social problem, an underground temperature control system has been developed. The temperature near the surface is controlled by flowing warm or cold water through pipes buried in the ground. In order to apply this control system to actual problems, several analyses and experiments for the ground temperature control system have been developed. Some papers [1–9] have been presented about the construction of the ground temperature control system being applied to the maintenance and management of the lawn. Adaptive and active controls include the Bang–Bang control, which involves turning the control system on and off, optimal control analysis by the Sakawa–Shindo method or the conjugate gradient method, predictive control, and real-time control based on fuzzy control theory. Considering the costs for operation of the system, it is difficult to apply continuous control of temperature at underground control points. Therefore, for practical uses, it is worth considering the possibility of Bang–Bang control.

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CCC 0271–2091/99/170261–14$17.50

There are still several problems to be solved, one of which is the treatment of the boundary conditions. The boundary conditions for the ground temperature control system are important. In past studies [1–9], to apply the control theory to the practical problem, the temperature of the ground surface was defined as a known value and used for the boundary condition in the computation. It has been treated that the temperature on the boundary is always known, in other words, the outside boundary values could be observed directly as a complete time function. Considering the control system for practical use, however, the observation area is restricted because of the many athletic activities on the field of the stadium. Therefore, to operate the system practically, the boundary condition of the ground surface must be determined within the limited measurement conditions. In the conventional way of treatment for the boundary condition, the Dirichlet condition in the computation has been adopted. However, in order to exactly simulate the phenomenon of heat transfer on the boundary, it is necessary to use heat flux. Introducing heat flux as a Neumann boundary condition, the treatment of solar radiation can be taken into account in the computation. In this case, it is necessary to identify the unknown parameters, such as the heat transfer coefficient and the solar radiation absorptivity, because the model of the heat flux includes these parameters. The reason why the Kalman filter and the extended Kalman filter [10] are applied in the present paper is to treat appropriately errors that exist in the observation and also in the discretization. In the observation, the data obtained include the instrumental and artificial errors. These errors are defined as observation noise. Therefore, it is desirable that observation data are not used directly to simulate the heat conduction. Thus, it is also necessary to perform analyses considering observation noise so as to remove observation errors. In the basic equation and the discretization, the errors also exist. These errors are defined as system noise. It is necessary to introduce the numerical procedure to consider these errors. The purpose of this study is to estimate the boundary condition for the calculation of the ground temperature control system by identifying the parameters.

2. HEAT CONDUCTION ANALYSIS

To obtain temperature, θ, by computation, the unsteady thermal conduction equation is introduced as follows:

$$\rho C_p \frac{\partial \theta}{\partial t} - \beta\left(\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2}\right) = Q, \tag{1}$$

where ρ is the density, C_p is the specific heat, β is the thermal conductivity, t is time, x and y are co-ordinates in space and Q is the heat production respectively. The basic equation (1) is accompanied by the following boundary and initial conditions:

Dirichlet boundary condition: $\theta = \hat{\theta}$ on S_1,

Newmann boundary condition: $q = \beta\left(\frac{\partial \theta}{\partial x} l + \frac{\partial \theta}{\partial y} m\right) = \hat{q}$ on S_2,

Initial condition: $\theta(t_0) = \hat{\theta}_0$ in V,

where the caret denotes the value specified on the boundary, the whole boundary S consists of boundaries S_1 and S_2, l and m are the direction cosine of the outward unit normal vector of the boundary S, V is the domain to be analyzed and q denotes heat flux. In this paper, heat flux, q, is assumed to consist of $\hat{q}_\alpha$ and $\hat{q}_\gamma$ as follows:

$$\hat{q} = \hat{q}_\alpha + \hat{q}_\gamma, \tag{2}$$

$$\hat{q}_\alpha = \alpha(\theta - \theta_c), \tag{3}$$

$$\hat{q}_\gamma = -\gamma g, \tag{4}$$

where α is the heat transfer coefficient, γ is the solar radiation absorptivity, θ_c is the open air temperature and g is the solar radiation. The basic equation (1) is discretized by the finite element method in space and by the Crank–Nicolson method in time based on the above boundary and initial conditions. The finite element matrix equivalent equation can be obtained as follows:

$$\left([M] + \frac{\Delta t}{2}[S]\right)\{\theta_{k+1}\} = \left([M] - \frac{\Delta t}{2}[S]\right)\{\theta_k\} + \Delta t\{Q_k\}, \tag{5}$$

where

$$M = \int_V \rho C_p(\Phi\Phi^T)\,\mathrm{d}V,$$

$$S = \int_V \beta(\Phi_{,x}\Phi_{,x}^T + \Phi_{,y}\Phi_{,y}^T)\,\mathrm{d}V,$$

$$Q = \int_S \alpha(\theta - \theta_c)\Phi\,\mathrm{d}S - \int_S \gamma g\Phi\,\mathrm{d}S,$$

and where Δt is the time increment, subscripted k are the time iterations. To compute the temperature distribution θ by Equation (5) it is necessary to know the coefficients α and γ and boundary condition $\hat{\theta}$.

3. KALMAN FILTER

The Kalman filter consists of system and observation equations:

System equation:

$$x_{k+1} = F_k x_k + G u_k; \tag{6}$$

Observation equation:

$$y_k = H_k x_k + v_k, \tag{7}$$

where x_k is the state value, which can not be observed directly; y_k is the observed value; F_k is the state transition matrix that is obtained by the coefficients of the finite element equation; H_k is the observation matrix, which has information about the placement of observation points; G is the driving matrix, which determines nodal points at which system noise u_k is taken into account; and v_k is the observation noise respectively. The noises u_k and v_k are assumed white noises, which are independent of each other and the averages are 0, i.e.

$$E\{u_k\} = 0, \qquad \operatorname{cov}\{u_k, u_j\} = E\{u_k u_j^T\} = Q\delta_{kj}, \tag{8}$$

$$E\{u_k\} = 0, \qquad \operatorname{cov}\{u_k, u_j\} = E\{u_k u_j^T\} = R\delta_{kj}, \tag{9}$$

$$\mathrm{cov}\{u_k, u_j\} = 0, \tag{10}$$

$$\delta_{ij} = \begin{cases} 1 & (i = j) \\ 0 & (i \neq j) \end{cases}, \tag{11}$$

where $E\{\ \}$ is the expectation operator and δ is the Kronecker delta.

To use the Kalman filtering technique, the following three conditions are postulated:

(i) The optimally estimated value $\hat{x}_k$ is the conditional average value on the condition that the observation data $Y_k = (y_0, y_1, \ldots, y_k)$ are given:

$$\hat{x}_k = E\{x_k | Y_k\}. \tag{12}$$

Therefore, the estimation error covariance P_k is represented as follows:

$$P_k = \mathrm{cov}\{x_k | Y_k\} = E\{(x_k - \hat{x}_k)(x_k - \hat{x}_k)^T\}. \tag{13}$$

(ii) The estimated value x_k^* is the conditional average value on condition that observation data $Y_k = (y_0, y_1, \ldots, y_{k-1})$ are given:

$$x_k^* = E\{x_k | Y_{k-1}\}. \tag{14}$$

Therefore, the prediction error covariance Γ_k is represented as follows:

$$\Gamma_k = \mathrm{cov}\{x_k | Y_{k-1}\} = E\{(x_k - x_k^*)(x_k - x_k^*)^T\}. \tag{15}$$

(iii) All processes of the Kalman filter complies with normal distribution, and the probability density function of the normal distribution is shown as:

$$p(x) = \frac{1}{(2\pi)^{1/2}|P_k|^{1/2}} \exp\left\{ -\frac{1}{2}(x - m)^T |P_k|^{-1}(x - m) \right\}, \tag{16}$$

where x is the probabilistic variable, m is the average value and P_k is covariance matrix respectively.

The probability density function of each condition is written as follows:

$$p(y_k|x_k) = N(Hx_k, R_k), \tag{17}$$

$$p(y_k | Y_{k-1}) = N(Hx_k^*, H\Gamma_k^* H^T + R_k), \tag{18}$$

$$p(x_k | Y_{k-1}) = N(x_k^*, \Gamma_k), \tag{19}$$

where $N(\ )$ denotes normal distribution. Using the Bayes rule,

$$p(x_k | Y_k) = \frac{p(y_k|x_k) p(x_k | Y_{k-1})}{p(y_k | Y_{k-1})}, \tag{20}$$

and employing all the above postulations, Equations (12), (13) and (15) are rewritten using the probability density function of the normal distribution. The estimated error covariance P_k, the prediction error covariance Γ_k and the optimally estimated value $\hat{x}_k$ can be expressed as follows:

$$\hat{x}_k = F_k \hat{x}_{k-1} + K_k(y_k - H_k F_k \hat{x}_{k-1}), \tag{21}$$

$$P_k = (I - K_k H_k)\Gamma_k, \tag{22}$$

$$\Gamma_{k+1} = F_k P_k F_k^T + G_k Q_k G_k^T, \tag{23}$$

where K_k is called as the Kalman-gain, which expresses the weight of the observation values at observation points against all other nodal points, and can be obtained as

$$K_k = \Gamma_k H_k^T (R_k + H_k \Gamma_k H_k^T)^{-1}. \tag{24}$$

The algorithm of the Kalman filter can be summarized as follows:

(1) $\Gamma_0 = V_0, \qquad x_0 = \hat{x}_0,$

(2) $K_k = \Gamma_k H_k^T (R_k + H_k \Gamma_k H_k^T)^{-1},$

(3) $P_k = (I - K_k H_k)\Gamma_k,$

(4) $\Gamma_{k+1} = F_k P_k F_k^T + G_k Q_k G_k^T,$

(5) $K_k - K_{k-1} > \epsilon,$ go to (2),

(6) $\hat{x}_k = F_k \hat{x}_{k-1} + K_k (y_k - H_k F_k \hat{x}_{k-1}).$

The Kalman filtering algorithm consists of two parts: the first part (1)–(5), the second part (6). The first part is called the 'on-line procedure' and the second part is called the 'off-line procedure'. In the off-line part, the calculation of the covariance matrix and the Kalman-gain matrix are iterated until the Kalman-gain converges. In the on-line part, the state vector is estimated using the Kalman-gain. The estimated value is updated gradually so that the renewed observation data can be included in the most useful way.

4. EXTENDED KALMAN FILTER

The extended Kalman filter, which is the linearized non-linear filter around the current mean and covariance, consists of system and observation equations as

System equations:

$$x_{k+1} = f_k(x_k) + u_k; \tag{25}$$

Observation equation:

$$y_k = h_k(x_k) + v_k, \tag{26}$$

where state value x_k is

$$x_k = (\theta \alpha \gamma)^T, \tag{27}$$

which are unknown parameters, and y_k denotes the observation vector, $f_k(x_k)$, $h_k(x_k)$ are non-linear functions of x_k. The system and observation noises are u_k and v_k respectively. The heat transfer coefficient α_k and solar radiation absorptivity γ_k are the discretized values at the time stage k. Because the non-linear functions in Equation (25) are continuous, Equation (25) can be linearized around the current estimate $\hat{x}_k$ using the Taylor series expansion as

$$f_k(x_k) = f_k(\hat{x}_k) + F_k(x_k - \hat{x}_k) + \cdots \ . \tag{28}$$

In Equation (28), F_k is

$$F_k = \left(\frac{\partial f_k}{\partial x_k}\right)_{x=\hat{x}_k} = \begin{bmatrix} \left(\frac{\partial \theta_{k+1}}{\partial \theta_k}\right)_{\theta=\hat{\theta}_k} & \left(\frac{\partial \theta_{k+1}}{\partial \alpha_k}\right)_{\alpha=\hat{\alpha}_k} & \left(\frac{\partial \theta_{k+1}}{\partial \gamma_k}\right)_{\gamma=\hat{\gamma}_k} \\ \left(\frac{\partial \alpha_{k+1}}{\partial \theta_k}\right)_{\theta=\hat{\theta}_k} & \left(\frac{\partial \gamma_{k+1}}{\partial \alpha_k}\right)_{\alpha=\hat{\alpha}_k} & \left(\frac{\partial \alpha_{k+1}}{\partial \gamma_k}\right)_{\gamma=\hat{\gamma}_k} \\ \left(\frac{\partial \gamma_{k+1}}{\partial \theta_k}\right)_{\theta=\hat{\theta}_k} & \left(\frac{\partial \gamma_{k+1}}{\partial \alpha_k}\right)_{\alpha=\hat{\alpha}_k} & \left(\frac{\partial \gamma_{k+1}}{\partial \gamma_k}\right)_{\gamma=\hat{\gamma}_k} \end{bmatrix}$$

$$= \begin{bmatrix} \left(\frac{\partial \theta_{k+1}}{\partial \theta_k}\right)_{\theta=\hat{\theta}_k} & \left(\frac{\partial \theta_{k+1}}{\partial \alpha_k}\right)_{\alpha=\hat{\alpha}_k} & \left(\frac{\partial \theta_{k+1}}{\partial \gamma_k}\right)_{\gamma=\hat{\gamma}_k} \\ 0 & [I] & 0 \\ 0 & 0 & [I] \end{bmatrix}. \tag{29}$$

Neglecting the higher-order terms in Equation (28), the linearized equation is obtained as follows:

$$x_{k+1} = F_k x_k + w_k + f_k(\hat{x}_k) - F_k \hat{x}_k, \tag{30}$$

where $\hat{x}_k$ is the conditional average value. Therefore, Equation (30) can be rewritten as

$$x_{k+1} = F_k x_k + w_k. \tag{31}$$

Equation (31) is almost in the same form as Equation (6), but F_k in Equation (31) is variable for each iteration cycle because F_k includes unknown parameters α_k and γ_k. Therefore, it is impossible to perform the calculation F_k by off-line computing.

The algorithm of the extended Kalman filter can be summarized as follows:

(1) $\Gamma_0 = V_0, \qquad x_0 = \hat{x}_0,$

(2) $K_k = \Gamma_k H_k^T (R_k + H_k \Gamma_k H_k^T)^{-1},$

(3) $P_k = (I - K_k H_k)\Gamma_k,$

(4) $\hat{x}_k = F_k \hat{x}_{k-1} + K_k(y_k - H_k F_k \hat{x}_{k-1}),$

(5) $\Gamma_{k+1} = F_k P_k F_k^T + G_k Q_k G_k^T,$

(6) Go to (2).

5. AR-MODEL

The state space model corresponding to a time series model is considered. The time series y_k obeys the autoregressive model as follows:

$$y_k = \sum_{i=1}^{m} a_i y_{k-i} + w_k. \tag{32}$$

The observation model can be obtained as follows by defining $H = [I \ \ 0 \cdots 0]$ because the first component in the state variable x_k is y_k as follows:

$$y_k = Hx_k, \tag{33}$$

therefore, x_k and x_{k-1} are related as follows, which is the same type of equation as Equation (6):

$$x_k = F_k x_{k-1} + G w_k, \tag{34}$$

where F and G can be expressed as the following $m \times m$ matrix and m dimensional vector

$$F_k = \begin{bmatrix} a_1 & a_2 & \cdots & a_m \\ I & & & \\ & \ddots & & \\ & & I & 0 \end{bmatrix}, \qquad G = \begin{bmatrix} I \\ 0 \\ \vdots \\ 0 \end{bmatrix}.$$

The algorithm of the Kalman filter for prediction is as follows:

One-step prediction:

$$x_k^* = F_k \hat{x}_{k-1}, \tag{35}$$

$$\Gamma_k = F_k P_{k-1} F_k^T + G_k Q_k G_k^T. \tag{36}$$

Filtering:

$$K_k = \Gamma_k H_k^T (R_k + H_k \Gamma_k H_k^T)^{-1}, \tag{37}$$

$$\hat{x}_k = x_k^* + K_k (y_k - H_k x_k^*), \tag{38}$$

$$P_k = (I - K_k H_k)\Gamma_k. \tag{39}$$

The time series $y_k = y_{k+1}$ is approved formally because the observed value y_{k+1} cannot be observed in the kth time step. Now, it is postulated as

$$\hat{x}_{k+1} = x_{k+1}^*, \tag{40}$$

$$P_{k+1} = \Gamma_{k+1}. \tag{41}$$

Two-step and long-term prediction are represented as follows:

Two-step prediction:

$$x_{k+2}^* = F_{k+2} \hat{x}_{k+1}, \tag{42}$$

$$\Gamma_{k+2} = F_{k+2} \Gamma_{k+1} F_{k+2}^T + G_{k+2} Q_{k+2} G_{k+2}^T. \tag{43}$$

Long-term prediction:

$$x_{k+j}^* = F_{k+j} \hat{x}_{k+j-1}, \tag{44}$$

$$\Gamma_{k+j} = F_{k+j} \Gamma_{k+j-1} F_{k+j}^T + G_{k+j} Q_{k+j} G_{k+j}^T, \quad j > 2. \tag{45}$$

In order to estimate the degree of the autoregression, the Akaike information criterion (AIC) is used as follows:

$$\mathrm{AIC}(J) = N(\log 2\pi\hat{\sigma}^2(J) + 1) + 2(J+1) \quad (J = 0, 1, \ldots, M). \tag{46}$$

AIC(0), ..., AIC(m), ..., AIC(M) are calculated and the optimal degree for the autoregression m is found by using the AIC. The degree m that minimizes the AIC(J) is the optimal degree for the autoregression, and M is the maximum degree.


Figure 1. Experimental site in Yokohama.

6. EXPERIMENTAL SITE

Figure 1 presents the experimental site in Yokohama city. Experimental observations are performed in this field for the duration February–June, 1997. The purpose of this experiment is to control underground temperature to be the appropriate condition for the lawn by flowing hot water (37°C) in pipes buried at -30 cm. Thermocouples were used to measure the temperature of the ground at -3, -5, -10, -15 and -30 cm respectively. In addition, open air temperature, solar radiation, humidity and rainfall are observed every 15 min.

The objective points of control are defined at -5 cm, which corresponds to the roots of lawn. In this analysis, the ground surface boundary condition is defined at -3 cm deep using the Dirichlet condition. Good agreement was obtained in the results of this experiment. The lawn in the controlled area was kept green compared with that in the non-controlled area. Figure 2 shows the condition of the lawn, the left side is the controlled area and the right side is the non-controlled area. Some parts of the lawn in the non-controlled area were dead. For the observation of temperature, it is confirmed that underground temperature in the controlled area, where hot water flowed in buried pipes, was kept about 5°C higher as compared with underground temperature in the non-controlled area. Figure 3 shows the observation data at the experimental field in Yokohama city from 27 May to 4 June 1997.

7. NUMERICAL EXAMPLES

One of the problems to be solved for practical usage is the treatment of boundary conditions. The observation of temperature near the ground surface for the boundary condition is almost impossible since many athletic activities are done on the field in the sport stadium. Therefore, to observe directly the temperature at -3 cm is almost impossible. For this reason, it is necessary that the boundary condition is predicted from the observation data within the

Figure 2. Controlled and non-controlled areas.

restricted observation range. Heat transfer values on the ground surface are influenced by various factors. To know the behavior of the heat transfer, heat flux must be introduced. As for the treatment of the boundary condition in this problem, the Neumann-type boundary was suitable compared with the Dirichlet condition. Adopting the Neumann condition for the ground surface boundary, the treatment of the solar radiation can be introduced. Introducing solar radiation, heat transfer on the ground surface can be described more clearly.

Using the heat flux, the heat transfer coefficient α and the solar radiation absorptivity γ should be known. Therefore, in this study the heat transfer coefficient and the solar radiation absorptivity are identified by the extended Kalman filter. Using these identified parameters, temperature at -3 cm has been determined. The temperature is used as the ground surface boundary condition for the calculation of the ground temperature control system by means of the standard Kalman filter.

For the identification, temperatures at -3, -5, -10, -15 and -30 cm, on the surface of the pipe, the solar radiation and the open air temperature are used as the observation data. For the estimation, identified parameters α and γ are used to describe the temperature model. Temperatures at -30 cm, the pipe, the open air temperature and the solar radiation are used as the observation data.

7.1. Identification of the heat transfer coefficient and the solar absorptivity

Identification of the heat transfer coefficient α and the solar radiation absorbtivity γ is performed with the extended Kalman filter and the finite element method. The calculation is iterated for the duration that the observation data are obtained.

In this study, relationships among α, γ and rainfall are investigated. Figure 4 shows the finite element mesh and arrangement of observation points. Table I shows the computation conditions. As for the computation terms, from 21 to 24 May and from 17 to 20 June, it rained more that 100 mm; from 26 to 29 May and from 12 to 15 June, it rained under 5 mm. Calculations for four cases are performed. Figure 5 shows the rainfall for a day. Table II

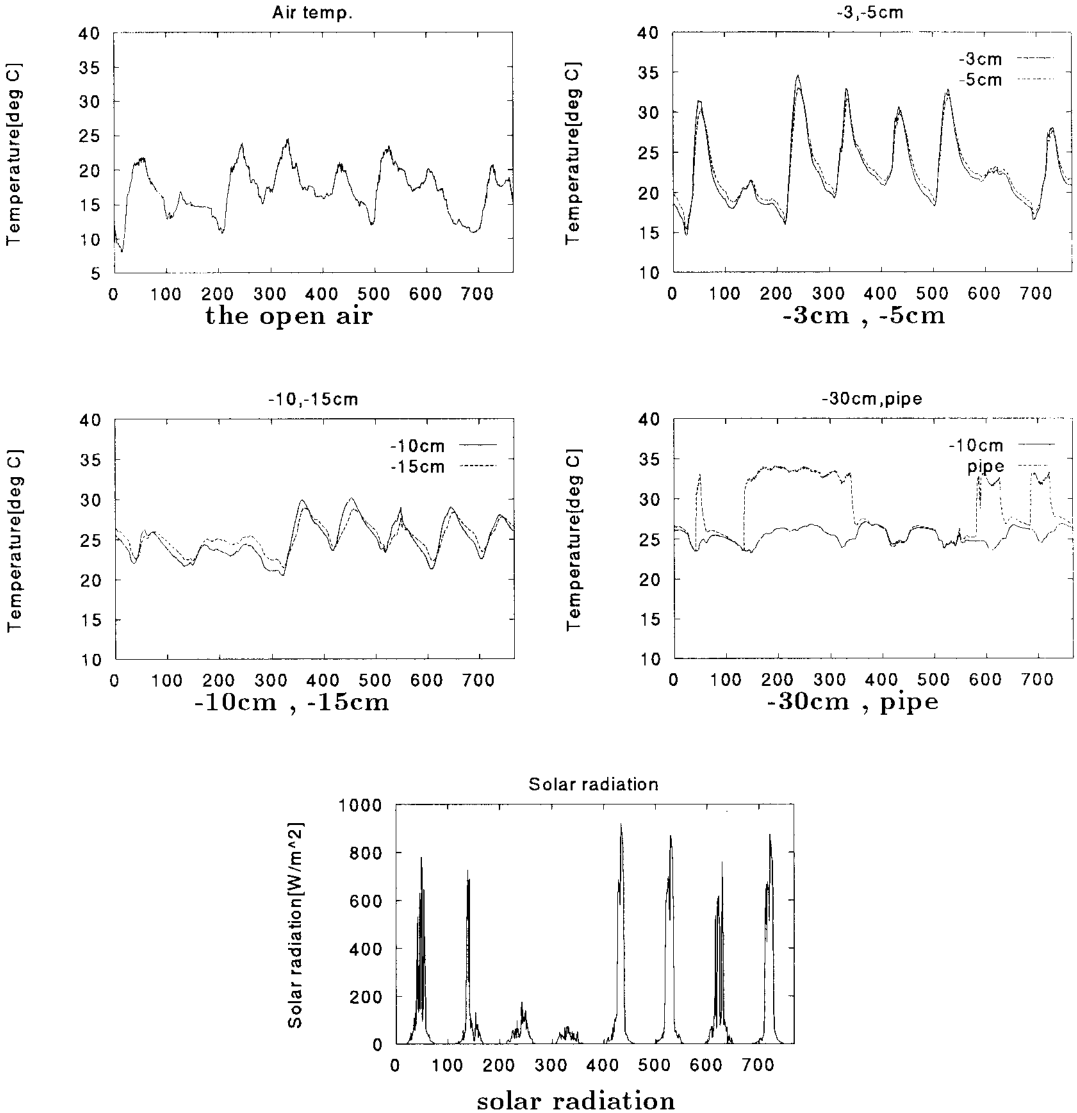

Figure 3. Observed data.

shows the identified heat transfer coefficient α and the solar radiation absorptivity γ. As a result of the identification, it is confirmed that α on rainy days is lower by about 10% than α on sunny days, whereas γ on sunny days is almost the same as γ on rainy days.

7.2. Estimation of boundary temperature

In this study, using the identified parameters, which are the heat transfer coefficient α and the solar radiation absorptivity γ in the former study, the estimation of the boundary condition for the ground temperature control system is performed by the standard Kalman filter and the finite element method. Open air temperature, temperature at -30 cm, temperature on the surface of the pipe and the solar radiation are used as the observation data. Figure 6 shows the

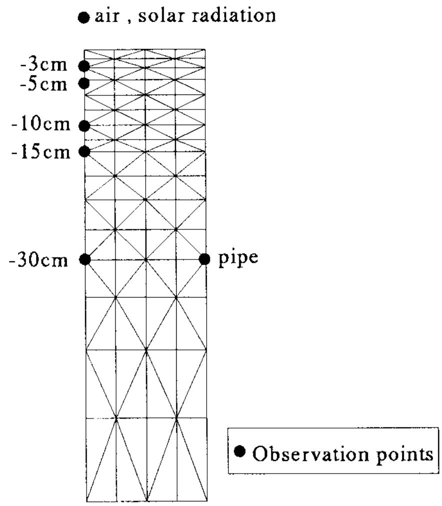

Figure 4. Finite element mesh.

Table I. Computation conditions

Δt	15.0 min
ρC_p	2000000.0
Thermal conductivity β_1	0.7
Thermal conductivity β_2	0.3
Thermal conductivity β_3	0.015
Initial value α_0	10.0
Initial value γ_0	0.8
System noise Q	0.05
Observation noise R	0.01

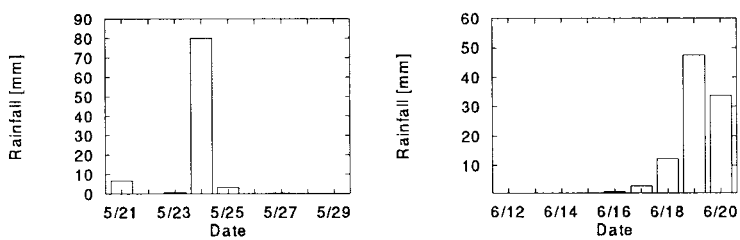

Figure 5. Rainfall.

Table II. Identified parameters

Terms	$\bar{\alpha}$	$\bar{\gamma}$
21–24 May (rainy days)	21.3	0.57
26–29 May (sunny days)	23.2	0.57
12–15 June (sunny days)	23.4	0.59
17–20 June (rainy days)	21.0	0.58

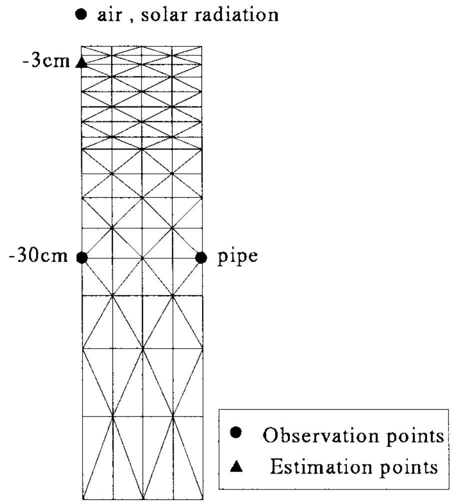

Figure 6. Finite element mesh.

finite element mesh, observation points and estimation points. Figure 7 shows the estimated temperature compared with the observed temperature at -3 cm deep. Both are in good agreement.

7.3. Prediction of boundary temperature

Moreover, using the estimated values in Figure 7, a prediction is carried out. The maximum degree of the autoregression M is 288, which is equal to 3 days. The prediction is started from the 289th step in Figure 8. Where Δt is 15 min, therefore the 12th, 24th, 48th and 96th step represent 3, 6, 12 and 24 h. The calculation is performed at every prediction step j in Equations (44) and (45), where j denotes the prediction for j steps ahead. In this analysis, calculations of prediction ($j = 12$, 24, 48, 96) are carried out. Figure 8 shows predicted values in this study and observed data that are observed directly in the experiment. In Figure 8, the result of prediction for 12, 24 and 48 steps ahead are good agreement with the observed values. Contrary to this, about 98 steps ahead, a change of temperature between daytime and night-time is not obtained.

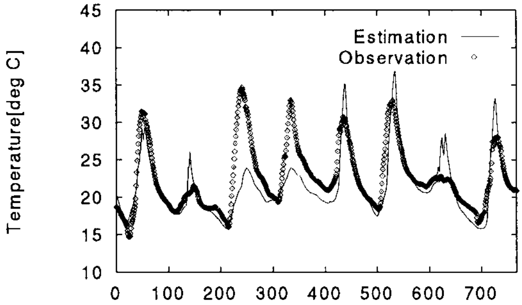

Figure 7. Estimated temperature.

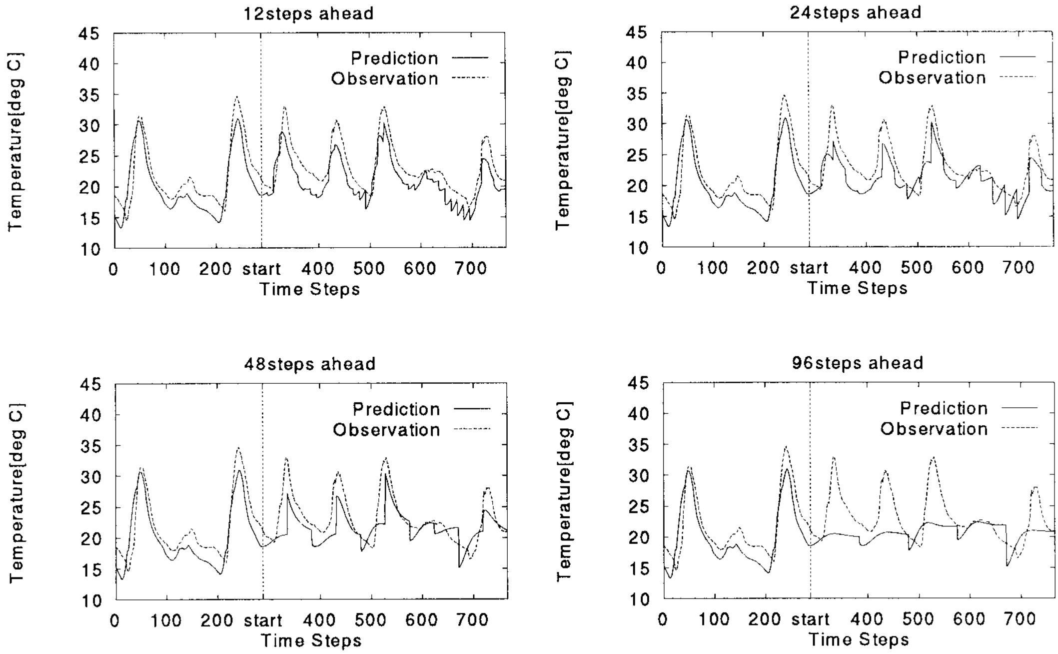

Figure 8. Predicted value.

8. CONCLUDING REMARKS

This paper has discussed (1) that the heat transfer coefficient α and the solar radiation absorptivity γ are identified by the extended Kalman filter and the finite element method and (2) that using these parameters, the boundary condition for the ground temperature control system is estimated and predicted by the Kalman filter and the finite element method. The Kalman filter is one of the estimation methods based on the stochastic process. The extended Kalman filter is also one of the identification methods based on the stochastic process. Calculations based on these methods are carried out by considering two kinds of noises: system and observation noises. It is very difficult to physically observe the data used in the calculation due to on going athletic activities in the stadium. It is nearly impossible to observe the temperature near the ground surface area (from 0 cm to -20 cm). Therefore, properties of the heat transfer coefficient and the solar radiation absorptivity must be computed almost the same way as applied to thermal conductivity β [1,2].

As for the estimation of the boundary condition, good estimated results are obtained within the limited observation area. The estimation using the standard Kalman filter is suitable for the on-line estimation for the operation of the ground temperature control system.

As for the prediction, the autoregressive model and the Kalman filter are applied. In the calculation of the autoregressive model, in order to determine the optimal degree m, the Akaike information criterion (AIC) is used. Numerical results are dependent on the maximum degree M, which is a total number of observation data for the calculation of the prediction and the optimal degree m that minimizes the value of AIC in large amounts. Therefore, the interaction between M and AIC(m) should be investigated thoroughly.

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